Properties of fluid

Dt-18.12.19

Chapter-1

Specific gravity $= \frac{\text{Density}}{\text{Specific gravity}} \times 100$

water = 1
Mercury = 13.6

Pressure in a liquid

$P = \rho g h$

specific weight
$= \frac{\text{weight}}{\text{unit volum}} = N/m^3$

Manometer

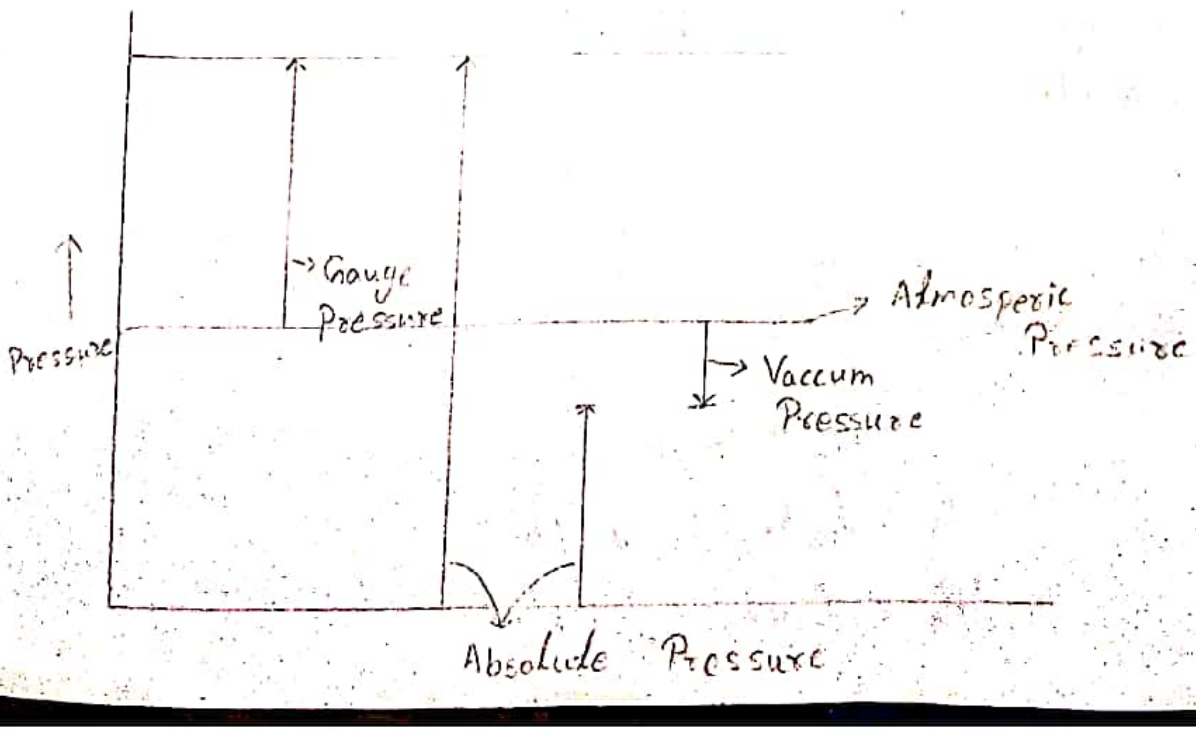
Dt 20.12.19

Simple manometer

- Piezometer
- simple U-tube manometer
- single coleumn Manometer

Differential manometer

- Differential U-tube manometer.
- Inverted U-tube manometer



$$P_{abs} = P_{atm} + P_{gauge}$$

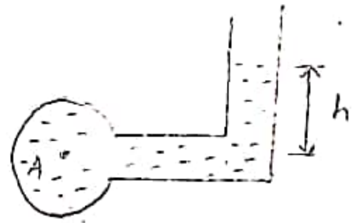
$$P_{abs} = P_{atm} - P_{vaccume}$$

Dt - 23.12.2019

(i) Piezometer

It is the simplest ~~form~~ form of manometer it consist of a glass tube where one end is connected to the point where the pressure is to the measure and other end remain open to the atmospere

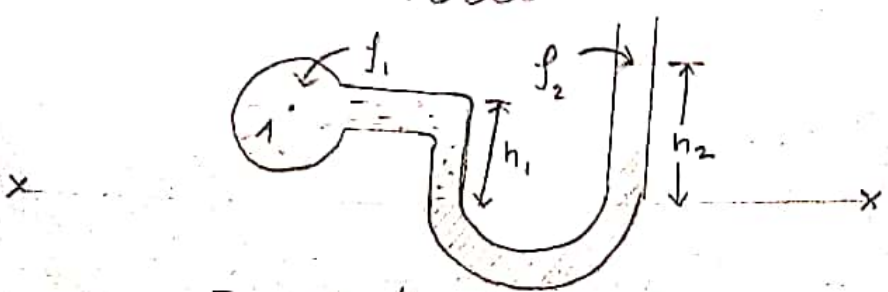
$$\boxed{P_A = \rho g h}$$



(ii) Simplest U-tube Manometer

It consist of a glass tube bend in U-shape here one end is connected to a point where the pressure is to be measured and other end remain open to the atmosphere in this type of manometer we can measured the pressure by ~~both~~ balancing the ~~two~~ two column of manometer. we take a liquid having specific gravity greater then the given liquid.

Case - 1

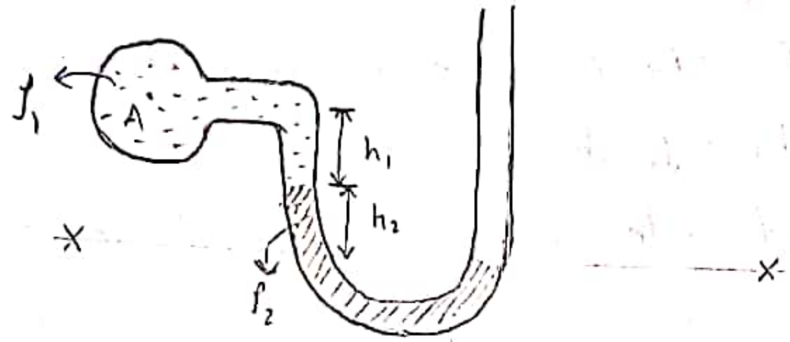


For Positive Gauge Pressure

$$P_A + \rho_1 g h_1 = \rho_2 g h_2$$

$$\Rightarrow P_A = \rho_2 g h_2 - \rho_1 g h_1$$

Case - 2



For Vacuum Pressure

$$P_A + \rho_1 g h_1 + \rho_2 g h_2 = 0$$

$$P_A = -(\rho_1 g h_1 + \rho_2 g h_2)$$

Let ρ_1 = density of the liquid whose pressure is to be measured

ρ_2 = density of heavy liquid

h_1 = rise of liquid in left limb

Case - 1

For gauge pressure

Total left side pressure is equal to

$P_A + \rho_1 g h_1$

Total right side pressure is equal to

$\rho_2 g h_2$

For free surface pressure is same at all the point so, left side pressure must be equal to right side pressure

$$P_A + \rho_1 g h_1 = \rho_2 g h_2$$

Case - II

For Vaccume Pressure

Total left side pressure is equal to

$P_A + \rho_1 g h_1 + \rho_2 g h_2$

Total right side pressure is equal to

0

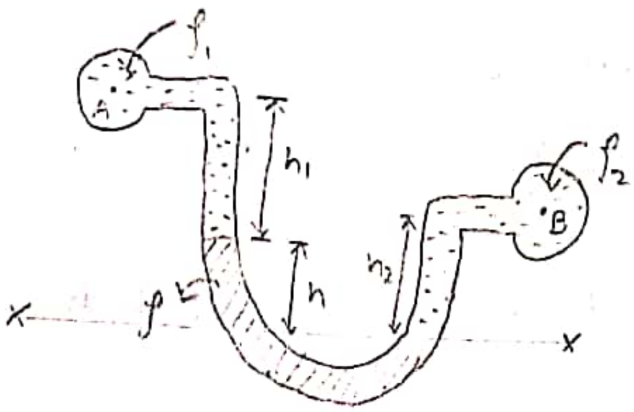
For free surface pressure is same at all the points. So, left side pressure must be equal with right side pressure.

$$P_A + \rho_1 g h_1 + \rho_2 g h_2 = 0$$

Dt - 07.01.2020

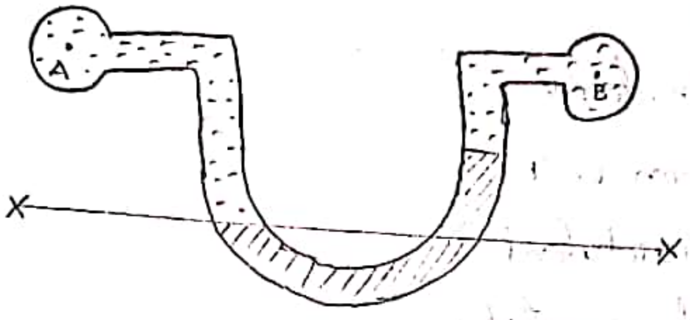
Differential U-tube manometer

It consist, it is used measure the difference of pressure in two points in a single pipe or in two different pipes.



$$P_A + \rho_1 g h_1 + \rho g h = P_B + \rho_2 g h_2$$

$$P_A - P_B = \rho_2 g h_2 - (\rho_1 g h_1 + \rho g h)$$



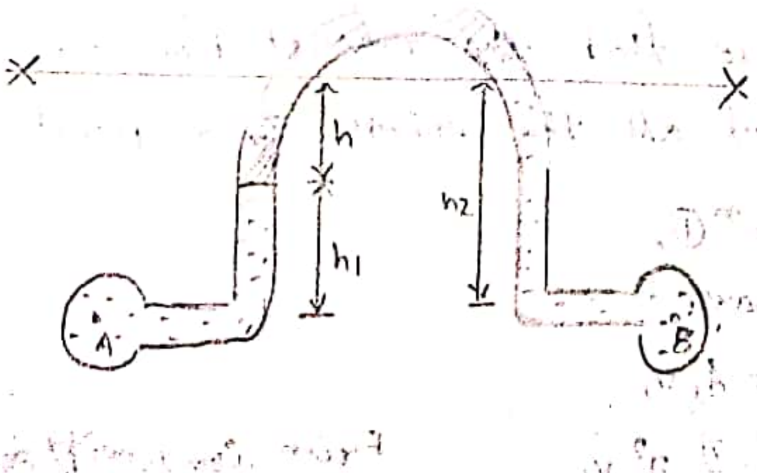
$$P_A + \rho_1 g h_1 = P_B + \rho_2 g h_2 + \rho g h$$

$$P_A - P_B = (\rho_2 g h_2 + \rho g h) - (\rho_1 g h_1)$$

Inverted U-tube manometer

$$P_A - \rho_1 g h_1 = P_B - \rho_2 g h_2 - \rho g h$$

$$P_A + P_B = \rho_1 g h_1 - (\rho_2 g h_2 + \rho g h)$$



$$P_A - \rho_1 g h_1 - \rho g h = P_B - \rho_2 g h_2$$

$$P_A + P_B = (\rho_1 g h_1 + \rho g h) - \rho_2 g h_2$$

Dt 10.01.2020

Types of Flow

① steady and unsteady

② Uniform and non uniform

③ Laminar and Turbulent

④ Rotational and Irrotational

⑤ Compressible and Incompressible

⑥ One, two and three dimentional

* All liquids are incompressible ($\rho = const$)
All gases are compressible ($\rho \neq const$)

Rate of flow or Discharge (Q)

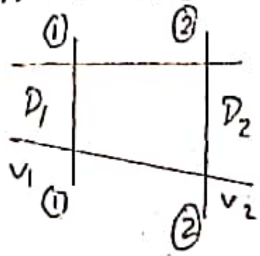
$$\boxed{Q = A \times V} \quad m^2\, m/s = \frac{m^3}{s}$$

$$\frac{m^3}{s} = 10^3 \times \frac{lit}{s}$$

It is the product of area and velocity. It is also known as volume rate of flow.

Continuity equation.

It state that the rate of flow or discharge is same at all the points. In a fluid flow



At secⁿ ①,

Discharge

$Q_1 = A_1 V_1$

$= \frac{\pi}{4} D_1^2 V_1$

$Q_2 = A_2 V_2$

$= \frac{\pi}{4} D_2^2 V_2$

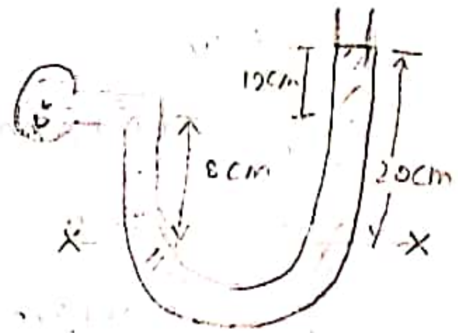
From continuity eqⁿ

$Q_1 = Q_2$

$$\boxed{A_1 V_1 = A_2 V_2}$$

Q. The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of specific gravity is 0.9 is flowing. The center of the pipe is 12 cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of the mercury level in the two limbs is 20cm

Given



specific gravity$_1$ = 0.9

$\rho_1 = 0.9 \times 1000 = 900\ kg/m^3$

$h_1 = 8cm = 0.08\ m$

specific gravity$_2$ = 13.6

$\rho_2 = 13600\ kg/m^3$

$h_2 = 20\ cm = 0.2\ m$

$p + \rho_1 g h_1 = \rho_2 g h_2$

$P = \rho_2 g h_2 - \rho_1 g h_1$

$= (13600 \times 9.81 \times 0.2) - (900 \times 9.81 \times 0.08)$

$= 26683.2 - 706.32$

$= 25976.88$

$= 25977\ N/m^2$

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Q A simple U-tube manometer containing mercury is connected to a pipe in which a fluid of sp.g = 0.8 and having vacuum pressure is flowing. The other end of the manometer is open to atmosper. Find the vacuum pressure in pipe. If the diffrance of mercury level in the two limb is 40 cm and the hight of fluid in the left from the center of pipe is 15 cm below.

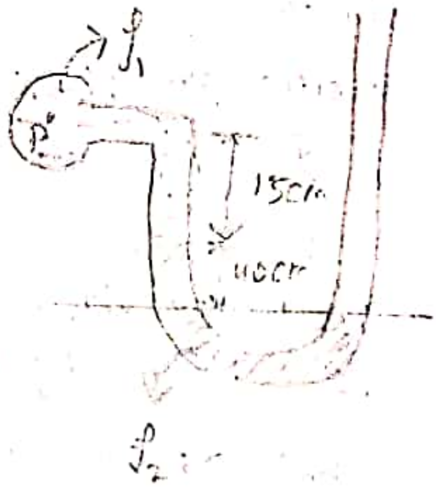
Given

$\rho_1 = 800 \ kg/m^3$

$h_1 = 0.15 \ m$

$\rho_2 = 1360 0 kg/m^3$

$h_2 = 0.4 \ m$



$$P + \rho_1 g h_1 + \rho_2 g h_2 = 0$$

$$P = -\left[(800 \times 9.81 \times 0.15) + (13600 \times 9.81 \times 0.4)\right]$$

$$= -54543.6 \ N/m^2$$

$$= -54544 \ N/m^2$$

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Dt 13.01.2020

Energies in a fluid flow (head) :

① Pressure energy / head $\left(\frac{P}{\rho g}\right)$

② Kinetic / velocity energy / head

$$V = \sqrt{2gh} \Rightarrow V^2 = 2gh \Rightarrow \boxed{h = \frac{V^2}{2g}}$$

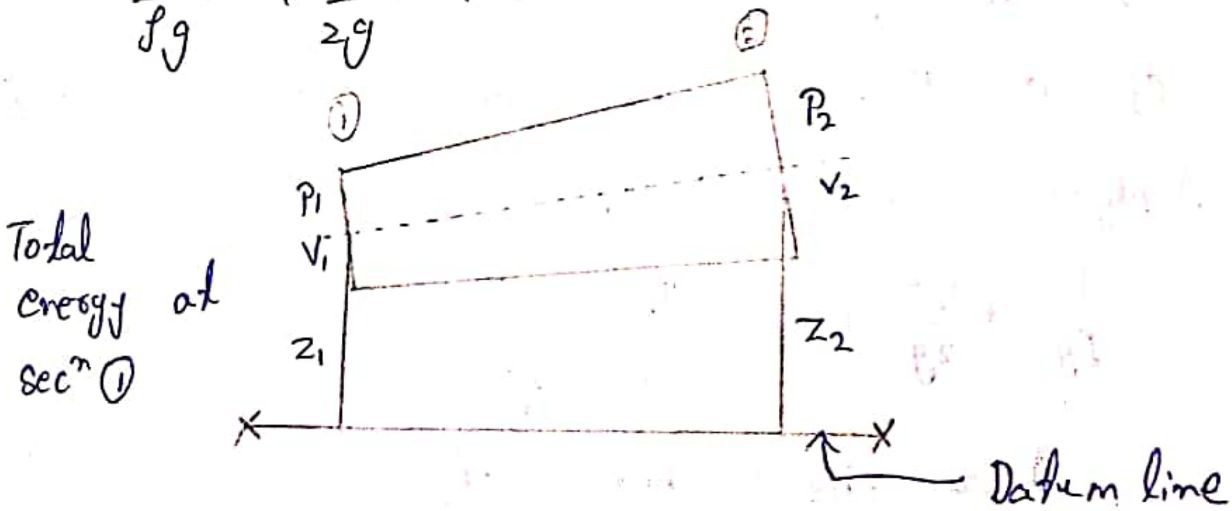
③ Datum / potential energy (z)

$$\text{Total energy} = \frac{P}{\rho g} + \frac{V^2}{2g} + z$$

Bernoulli's Theorm

It state that the total energy is same at all the points in a fluid flow.

$$\frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{const}$$



$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

Practical application of Bernoulli's equation

① Venturimeter ② Orificemeter

③ Pitot tube

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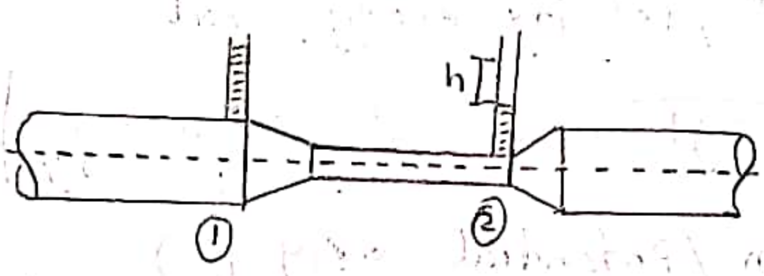
① Venturimeter

It is a device used to measure the rate of flow in a pipe it consist of one converging end, one diverging end and a throat.



Consider two sections, section ① at pipe and section ② at throat. Let

d_1 = diameter of pipe at section ①

a_1 = area of pipe at section ①

$= \frac{\pi d_1^2}{4}$

P_1 = Pressure at section ①

V_1 = Velocity at section ①

d_2, a_2, P_2, V_2 = Corresponding value at sec^n ②

Applying Bernoulli's eq^n at sec^n ①

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

As the pipe is horizontal

$Z_1 = Z_2$

So,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g}$$

$$h = \frac{V_2^2 - V_1^2}{2g}$$ Let $\frac{P_1 - P_2}{\rho g} = h$

$$2gh = V_2^2 - V_1^2 \quad \text{———} \quad (i)$$

Now appling continuety eqn at secn ① and ②

$$a_1 V_1 = a_2 V_2$$

$$V_1 = \frac{a_2 V_2}{a_1}$$

Now putting value of V_1 in eqn (i)

$$2gh = V_2^2 - \left(\frac{a_2 V_2}{a_1}\right)^2$$

$$2gh = V_2^2 - \frac{a_2^2 V_2^2}{a_1^2}$$

$$2gh = V_2^2 \left(1 - \frac{a_2^2}{a_1^2}\right)$$

$$2gh = V_2^2 \left(\frac{a_1^2 - a_2^2}{a_1^2}\right)$$

$$V_2^2 = 2gh \left(\frac{a_1^2}{a_1^2 - a_2^2}\right)$$

$$V_2 = \sqrt{2gh \left(\frac{a_1^2}{a_1^2 - a_2^2}\right)}$$

$$V_2 = \sqrt{2gh}\left(\frac{a_1}{\sqrt{a_1^2 - a_2^2}}\right)$$

Now discharge

$$Q = a_2 V_2$$

$$= a_2 \times \sqrt{2gh}\left(\frac{a_1}{\sqrt{a_1^2 - a_2^2}}\right)$$

$$\boxed{Q = \sqrt{2gh}\,\frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}}}$$

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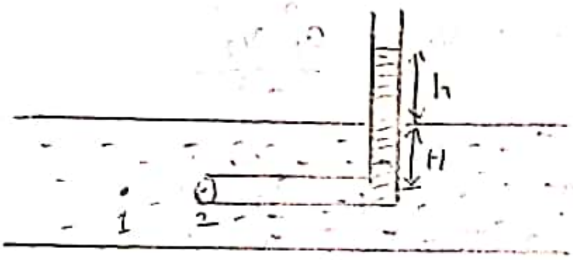
$$Q_{actual} = C_d \sqrt{2gh} \left(\frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \right)$$

where

C_d = Coefficient of discharge it's value is generaly taken as 0.98.

Pitot Tube

It is a device used to measure the rate of flow in a Pipe. It consist of a glass tube bend in as shown in the figure



Consider point 1 in side the pipe and point 2 just at the inlet of the pitot tube

h = Rise of liquid above the surface of pipe

H = Depth of pitot tube inside the liquid

Let

P_1 = Pressure at point one

V_1 = velocity at point one

P_2, V_2 = Corresponding value at point two

Now appling bernoulli's equation at point ① and ②

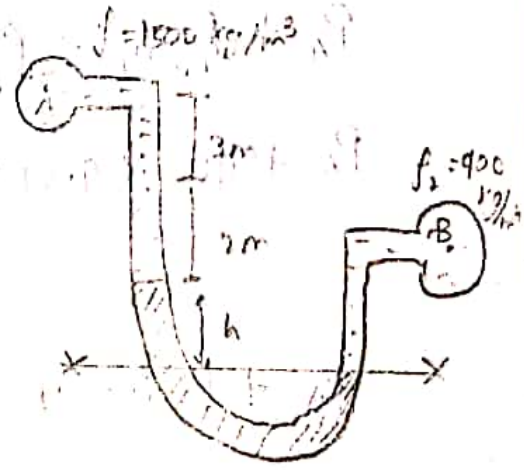
$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$H + \frac{V_1^2}{2g} = H + h + 0$$

$$v_1^2 = 2gh$$
$$v_1 = \sqrt{2gh}$$
$$V_{actual} = C_v \sqrt{2gh}$$
C_v = coefficient of velocity.

Dt 14.01.2020

Q A differential manometer is connected at two point A and B of two pipes as shown in the figure the pipe A contain a liquid of specific gravity 1.5 while the pipe B contain a liquid of specific gravity 0.9 the pressures at A and B are 1 kgf/cm² and 1.8 kgf/cm² find the difference in mercury level in the differential manometer



$\rho = 13600\ kg/m^3$

$$P_A = 1\ kgf/cm^2$$
$$= 9.81 \times 10^4\ N/m^2$$
$$P_B = 1.8 \times 9.81 \times 10^4\ N/m^2$$

$$P_A + \rho_1 g h_1 + \rho g h = P_B + \rho_2 g h_2$$

$$9.81 \times 10^4 + 1500 \times 9.81 \times 5 + 13600 \times 9.81 \times h$$
$$= 1.8 \times 9.81 \times 10^4 + 900 \times 9.81 \times (2+h)$$

$$171675 + 133416h = 176580 + (8829(2+h))$$

$$133416h - 8829h = 176580 + 17658 - 171675$$
$$124587h = 22563$$

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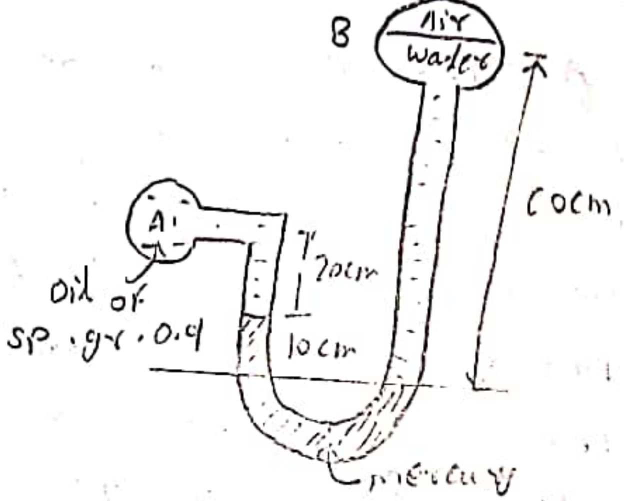
$$h = \frac{22563}{124587}$$

$$= 0.18 \text{ m}$$

A differential manometer is connected at the point A and B as shown in the fig. At B air pressure is 9.81 N/cm² . Find the absolute pressure at A



$$P_A + \rho_1 g h_1 + \rho g h = P_B + \rho_2 g h_2$$

$$P_A + 900 \times 9.81 \times 0.2 + 13600 \times 9.81 \times 0.1$$

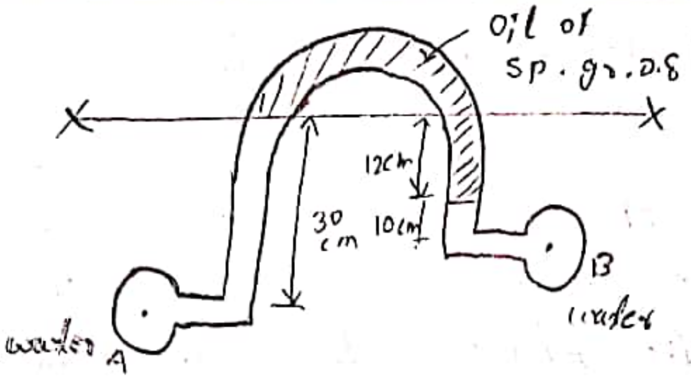
$$= 9.81 \times 10^4 + 1000 \times 9.81 \times 0.6$$

$$P_A + 15107.4 = 103986$$

$$P_A = 103986 - 15107.4$$

$$= 88878.6 \text{ N/m}^2.$$

Q water is flowing through two different pipe an inverted differential manometer having an oil of sp. gr. 0.8 is connected the pressure head in pipe A is 2m of water find the pressure in pipe B as shown in the fig.



$h = 2$

$h = \frac{P_A}{\rho g} \Rightarrow P_A = \rho g h$

$= 1000 \times 9.81 \times 2$

$= 19620 \ N/m^2$

$19620 - (1000 \times 9.81 \times 0.3) = P_B - (1000 \times 9.81 \times 0.1) - (800 \times 9.81 \times 0.12)$

$16677 = P_B - 1922.76$

$P_B = 16677 + 1922.76$

$= 18599.76 \ N/m^2$

<u>Specific volume</u>

It is defined as the ratio of the volume of a fluid to it's mass

$\text{sp. vol} = \frac{\text{Volume of fluid}}{\text{mass of fluid}}$

$= \frac{1}{\frac{\text{mass of fluid}}{\text{vol of fluid}}} = \frac{1}{\rho}$

<u>viscosity</u>

It is defined as the property of a fluid which offeres resistance to the moment of 1 layer of fluid to the adjusent layer of the fluid.

Dt:- 17.01.2020

Q The diameters of a pipe at the section 1 and 2 are 10cm and 15cm respectively. Find the discharge through the pipe if the velocity of the water flow through the pipe at section 1 is 5 m/s determine also the velocity at section 2

Ans Given

$$A_1 = \frac{\pi d^2}{4} = \frac{\pi \times (0.1)^2}{4} = 7.854 \times 10^{-3}$$

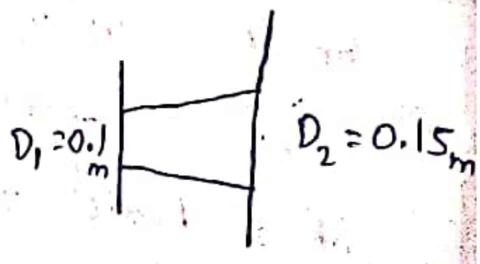
$$A_2 = \frac{\pi d^2}{4} = \frac{\pi \times (0.15)^2}{4} = 0.0176$$

$$V_1 = 5 \text{ m/s}$$

~~$A_1 V_1 = A_2 V_2$~~

$$Q = A_1 V_1$$
$$= 0.03927 \text{ m}^3/\text{s}$$

$$Q = A_2 V_2$$



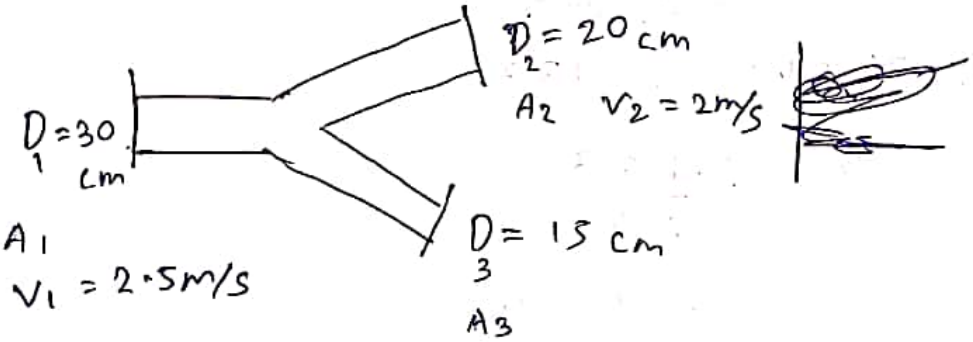
$$0.03927 = 0.0176 \times V_2$$

$$V_2 = \frac{0.03927}{0.0176}$$

$$= 2.23125 \text{ m/s}$$

$$= 2.23 \text{ m/s}$$

Q A 30 cm Diameter pipe conveying water branches into two pipe of diameter 20 cm and 15 cm respectively if the avarage velocity in 30 cm diameter pipe is 2.5 m/s find the discharge in this pipe also determine the velocity in 15 cm pipe ~~and~~ if the avarage velocity in 20 cm pipe is 2 m/s



$$A_1 = \frac{\pi d^2}{4} = \frac{\pi \times (0.3)^2}{4} = 0.0706 \text{ m}$$

$$A_2 = \frac{\pi d^2}{4} = \frac{\pi \times (0.2)^2}{4} = 0.0314 \text{ m}$$

$$A_3 = \frac{\pi d^2}{4} = \frac{\pi \times (0.15)^2}{4} = 0.0176 \text{ m}$$

$$Q_1 = A_1 V_1$$
$$= 0.0706 \times 2.5$$
$$= 0.1765 \text{ m}^3/\text{s}$$

$$Q_2 = A_2 V_2$$
$$= 0.0314 \times 2$$
$$= 0.0628 \text{ m}^3/\text{s}$$

$$Q_1 = Q_2 + A_3 V_3$$

$$0.1765 = 0.0628 + 0.0176 \times V_3$$

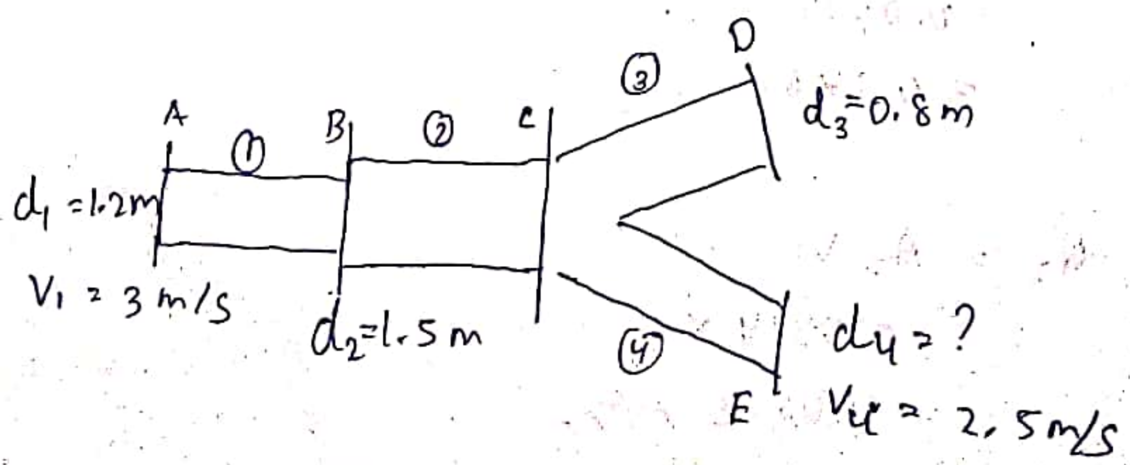
$$0.0176 \times V_3 = 0.1765 - 0.0628$$

$$0.0176 \times V_3 = 0.1137$$

$$V_3 = \frac{0.1137}{0.0176}$$

$$= 6.46 \text{ m/s}$$

Q water flows through a pipe A ~~and~~ B 1.2 m diameter, at ~~and~~ 3m/s and then passes through a pipe BC ~~B and~~ 1.5 m diameter at C the pipe branches. Branch CD is 0.8 m in diameter and carries 1/3 of the flow in AB the flow velocity in branch CB is 2.5 m/s find the volume rate of flow in AB, the velocity in BC, the velocity in CD and diameter of 'DE'



$A_1 = \frac{\pi d^2}{4} = 1.1309\ m^2$

$A_2 = \frac{\pi d^2}{4} = 1.7671\ m^2$

$A_3 = \frac{\pi d^2}{4} = 0.5026\ m^2$

$Q_1 = A_1 V_1$

$= 3.3927\ m^3/s$

$\boxed{Q_1 = Q_2}$

$Q_2 = A_2 V_2$

$3.3927 = 1.7671 \times V_2$

$V_2 = \frac{3.3927}{1.7671}$

$= 1.9199\ m/s$

$Q_1 \times \frac{1}{3} = Q_3$

$Q_3 = \frac{3.3927}{3} = 1.1309\ m^3/s$

$Q_3 = A_3 V_3$

$1.1309 = 0.5026 \times V_3$

$V_3 = \frac{1.1309}{0.5026} = 2.25\ m/s$

$Q_2 = Q_3 + A_4 V_4$

$3.3927 = 1.1309 + A_4 \times 2.5$

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$$A_4 \times 2.5 = 3.3927 - 1.1309$$

$$A_4 = \frac{2.2618}{2.5}$$

$$= 0.90472 \ m^2$$

$$A_4 = \frac{\pi d^2}{4}$$

$$0.9 \times 4 = \pi d^2$$

$$\pi d^2 = 3.6$$

$$d^2 = \frac{3.6}{\pi}$$

$$d = \sqrt{1.1459}$$

$$= 1.0704 \ m$$

Dt 21.01.2020

Q) water is flowing throughing a pipe of 5cm diameter under a pressure of 29.43 N/cm^2 and with mean velocity of 2 m/s find the total head or total energy per unit weight of water at a cross section which is 5m above the datum line

given

$P = 29.43 \ N/cm^2 = 29.43 \times 10^4 \ N/m^2$

$v = 2 \ m/s$ $\rho = 1000$

$z = 5 \ m$

$$\frac{P}{\rho g} = \frac{29.43 \times 10^4}{1000 \times 9.81} = 30 \text{ m}$$

$$\frac{V^2}{2g} = \frac{(2)^2}{2 \times 9.81} = 0.2038 \text{ m}$$

$Z = 5$ m

Total head

$= 30 + 0.2038 + 5$

$= 35.2038$ M

Q A pipe through which water is flowing is having diameter 20 cm and 10 cm at the cross sections 1 and 2 respectively the velocity of water at section 1 is given 4 m/s find the velocity head at section 1 and 2 and also the rate of discharge..

$$A_1 = \frac{\pi d^2}{4} = \frac{\pi (0.2)^2}{4}$$

$$= 0.0314$$

$$A_2 = \frac{\pi d^2}{4} = \frac{\pi (0.1)^2}{4} = 7.8539 \times 10^{-3}$$

$A_1 V_1 = A_2 V_2$

$0.0314 \times 4 = 7.853 \times 10^{-3} \times V_2$

$$v_2 = \frac{0.0314 \times 4}{7.853 \times 10^{-3}}$$

$= 15.99$

$= 16 \ m/s$

The rate of discharge

$Q = 0.1256 \ m^3/s$

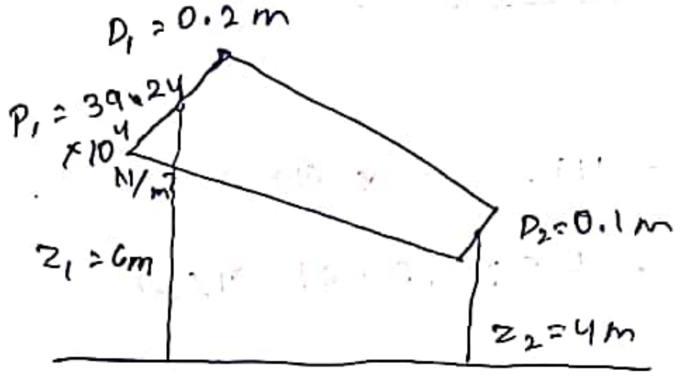
velocity head (1)

$$\frac{v^2}{2g} = \frac{(4)^2}{2 \times 9.81} = 0.8015 \ m$$

velocity head (2)

$$\frac{v^2}{2g} = \frac{(16)^2}{2 \times 9.81} = 13.047 \ m$$

Q The water is flowing through a pipe of diameters 20 cm and 10cm at section 1 and 2 respectively. The rate of flow through the pipe is 35 L/s the section 1 is 6m above the datum line and section 2 is 4m above the datum line. If the pressure at section 1 is 39.24 N/cm² find the intensity of pressure at section 2.



$Q = 35\ lit/s$

$= 0.035\,m^3/s$

By appling Bernoulli's eqn

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

By appling continuty eqn

$Q = A_1 V_1 = A_2 V_2$

$$V_1 = \frac{Q}{A_1} = \frac{0.035}{\frac{\pi \times (0.2)^2}{4}} = 1.114\ m/s$$

$$V_2 = \frac{Q}{A_2} = \frac{0.035}{\frac{\pi \times (0.1)^2}{4}} = 4.456\ m/s$$

$$= \frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 6 = 46.063$$

$$= \frac{P_2}{1000 \times 9.81} + \frac{(4.456)^2}{2 \times 9.81} + 4 = 5.012 + \frac{P_2}{9810}$$

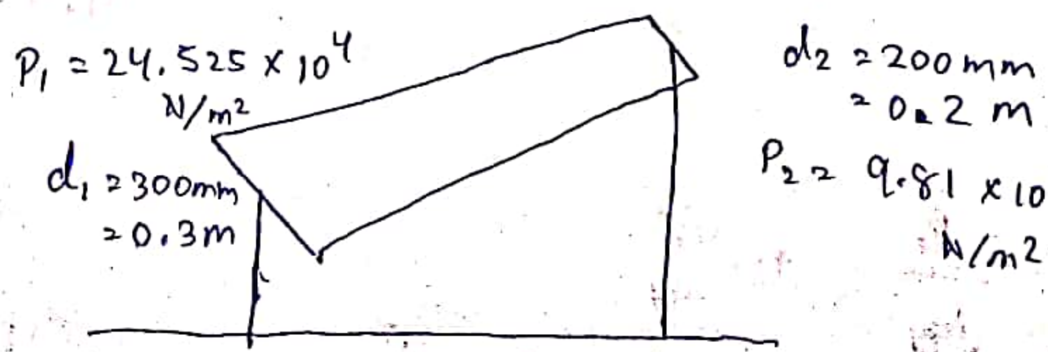
$$\frac{P_2}{9810} = 46.063 - 5.012$$

$$P_2 = 41.051 \times 9810$$

$$= 402710.31 \ N/m^2$$

$$= 40.271031 \ N/cm^2$$

Q water is flowing through a pipe having diameter 300 mm and 200 mm at the bottom and upper end respectively the intensity of pressure at the bottom end is 24.525 N/cm² and the pressure at the upper end is 9.81 N/cm² determine the difference in datom head if the rate of flow through pipe is 40 lit per sec



$$Q = 40 \ lit/s$$

$$= 0.04 \ m^3/s$$

$$V_1 = \frac{Q}{A_1}$$

$$= \frac{0.04}{\frac{\pi \times (0.3)^2}{4}} = 0.565 \ m/s$$

$$V_2 = \frac{Q}{A_2}$$

$$= \frac{0.04}{\frac{\pi \times (0.2)^2}{4}} = 1.273 \ m/s$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{24.525 \times 10^4}{1000 \times 9.81} + \frac{(0.565)^2}{2 \times 9.81} + Z_1 = \frac{9.81 \times 10^4}{1000 \times 9.81} + \frac{(1.273)^2}{2 \times 9.81} + Z_2$$

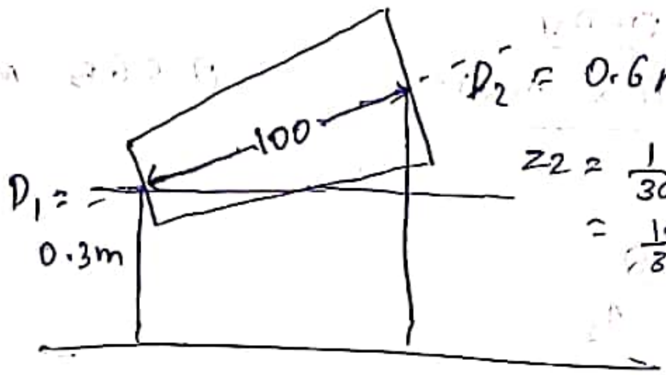
$$25.016 + Z_1 = 10.082 + Z_2$$

$$Z_2 - Z_1 = 25.016 - 10.085$$

$$= 14.931 \ m$$

24. 01. 2020

water is flowing through a tapper pipe of length 100 m having diameters 600 mm at the upper and and 300 mm at the lower end at a rate of 50 lit /sec the pipe has a slope of 1 in 30 find the pressure at the lower end if the pressure at the higher level is 19.62 N/cm²



$z_2 = \frac{1}{30} \times 100$

$= \frac{10}{30}$

$Q = 0.05 \, m^3/s$

$P_2 = 19.62 \times 10^4 \, N m^{-2}$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$V_1 = \frac{Q}{A_1}$

$= \frac{0.05}{\frac{\pi (0.3)^2}{4}} = 0.7073 \, N/m^2$

$V_2 = \frac{Q}{A_2}$

$= \frac{0.05}{\frac{\pi (0.6)^2}{4}} = 0.1768 \, N/m^2$

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$$\frac{P_1}{1000 \times 9.81} + \frac{(0.7073)^2}{2 \times 9.81} + 0$$

$$= \frac{P_1}{9810} + 0.0254$$

$$\frac{19.62 \times 10^4}{1000 \times 9.81} + \frac{(0.1768)^2}{2 \times 9.81} + \frac{10}{3}$$

$$= 23.3349$$

$$\Rightarrow \frac{P_1}{9810} = 23.3349 - 0.0254$$

$$\Rightarrow \frac{P_1}{\cancel{9810}} = \frac{23.3349 - 0.0254}{\cancel{9810}} \times 9810$$

$$= 23.3095 \times 9810$$

$$= 228666.195$$

$$= 22.8666 \ N/cm^2$$

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~~Case~~ Value of H in Venturimeter

Case - I

If the differancial manometer contains a liquid ~~water~~ heaver then the liquid flowing through the pipe

Let

S_h = Specific gravity of the heaver

S_o = Specific gravity of liquid flowing through the pipe

x = difference of heaver liquid coloumn in U - tube

$$\boxed{h = x\left[\frac{S_h}{S_o} - 1\right]}$$

Case - II

Let S_L = Specific gravity of the light liquid

S_o = Specific gravity of liquid flowing through the pipe

x = differance of light liquid column in U - tube

$$\boxed{h = x\left[1 - \frac{S_l}{S_o}\right]}$$

Q A horizontal venturimeter with inlet and throat diameter 30 cm and 15 cm is used to measure the flow or water the reading of differencial manometer connected to the inlet and throat is 20 cm of mercury determine the rate of flow.

$\boxed{C_d = 0.98}$

$d_1 = 0.3\ m$ $S_h = 13.6$

$d_2 = 0.15\ m$ $S_o = 1$

$x = 0.2\ m$

$$h = x\left[\frac{S_h}{S_o} - 1\right]$$

$$= 0.2\left[\frac{13.6}{1} - 1\right]$$

$$= 2.52\ m$$

$$Q_{act} = C_d \frac{a_1 \cdot a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$a_1 = \frac{\pi d_1^2}{4} = 0.0706\ m^2$

$a_2 = \frac{\pi d_2^2}{4} = 0.0176\ m^2$

$$Q_{act} = 0.98 \frac{(0.0706)(0.0176)}{\sqrt{(0.0706)^2 - (0.0176)^2}} \sqrt{2 \times 9.81 \times 252}$$

$= 0.1252 \ m^3/s$

Q an oil of sp. gr = 0.8 is flowing through a venturi meter having a inlet diameter 20cm d_2 = 10 cm the oil mercury manometer shows a reading of 25 cm . calculate the discharge of oil through the horizontal venturimeter

$S_h = 13.6$ $d_1 = 0.2$

$S_o = 0.8$ $d_2 = 0.1$

$x = 0.25$

$$a_1 = \frac{\pi d_1^2}{4} = \frac{\pi \times (0.2)^2}{4} = 0.0314$$

$$a_2 = \frac{\pi d_2^2}{4} = \frac{\pi \times (0.1)^2}{4} = 7.85 \times 10^{-3}$$

$$h = x \left[\frac{S_h}{S_0} - 1 \right]$$

$$= 0.25 \left[\frac{13.6}{0.8} - 1 \right]$$

$= 4$

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$$Q_{act} = 0.98 \times \frac{(0.0314)(7.85\times10^{-3})}{\sqrt{(0.0314)^2 - (7.85\times10^{-3})^2}} \times \sqrt{2\times9.81\times4}$$

$$= 0.0703 \ m^3/s$$

Dt 01.02-2020

Q A pitost static tube placed in the centre of 300mm pipe line has one oriphise pointing upstrem and other ter to it the mean velocity in the pipe is 0.8 of the central velocity find the discharge through the pipe if the pressure diffrence between the two orifise is 60 mm of water take Cv = 0.98

Given

$d = 0.3 m$ $\qquad C_v = 0.98$

$h = 0.06 m$

$\bar{V} = 0.8 \times V$

$V = C_v \sqrt{2gh}$

$V = 0.98 \sqrt{2\times9.81\times0.06}$

$= 1.063 \ m/s$

$\bar{V} = 0.8 \times V$

$= 0.8 \times 1.063$

$= 0.850 \ m/s$

~~Ans.~~

$Q = A\bar{V}$

$= \frac{\pi}{4} d^2 \times 0.850$

$= 0.07 \times 0.850$

$= 0.0595 \ m^3/s$

Q find the velocity of the flow of an oil through a pipe when the difference in mecreury level in a differential manometer connected to the two tappings of the pitot tube is 100 mm taken Cv = 0.98 and specific gravity of oil is 0.8

Given

$x = 0.1 m$

$C_v = 0.98$

$S_h = 13.6$, $S_o = 0.8$

$h = x\left[\frac{S_h}{S_o} - 1\right]$

$\boxed{h = 1.6}\ m$

$V = C_v\sqrt{2gh}$

$= 0.98 \times \sqrt{2 \times 9.81 \times 1.6}$

$= 5.4907 \ m/s$

Q A pitot tube is inserted in a pipe of 300mm diameter the static pressure in the pipe is 100mm of mercury (vacuum) the stagnation pressure at the center of the pipe recorded by the pitot tube is 0.981 N/cm² calculate the rate of flow of water through pipe if the mean velocity of flow is 0.85 times the central velocity taken $C_v = 0.98$

<u>Given</u>

$d = 300\ mm$

static pressure head $= -100\ mm\ of\ Hg$

$= -0.1\ m\ of\ Hg$

$= -0.1 \times 13.6\ m\ of\ water$

$= -1.36\ m\ of\ water$

stagnation pressure $= 0.981 \times 10^4\ N/m^2$

stagnation pressure head $= \dfrac{0.981 \times 10^4}{1000 \times 9.81} = 1\ m$

$h = 1 + 1.36$

$= 2.36\ m$

$\bar{V} = 0.85 \times V$

$= 5.6678\ m/s$

$V = C_v \sqrt{2gh}$

$= 0.98 \times \sqrt{2 \times 9.81 \times 2.36}$

$= 6.668\ m/s$

$Q = A \times \bar{V} = 0.0706 \times 5.6678$

$= 0.4\ m^3/s$

Hydrostatics

Total pressure

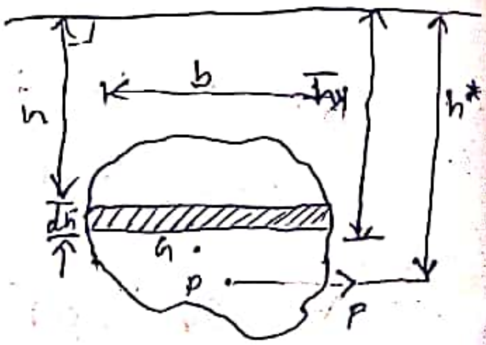
It is defined as the force exerted by a static fluid on a surface either plane or curved when the fluid comes in contact with the surface. This force always act normal to the surface.

Center of pressure

It is defined as the point of application of the total pressure on the surface

→ vertical plane surface submerged in a liquid

Consider a plane vertical surface of arbitary shape immersed in a liquid



Let

A = total area of the surface

$\bar{h}$ = distance of CG of the area from free surface of liquid

G = center of gravity of the plane surface

P = center of pressure

h^* = distance of center of pressure from the free surface of liquid.

Total pressure (P)

consider a strip of thickness 'dh' and width 'b' at a depth of 'h' from the free surface of liquid

pressure intensity on the strip

$$p = \rho g h$$

Area of the strip

$$dA = b \times dh.$$

Total pressure force on strip :

$$dF = p \times dA$$

$$= \rho g h \times b \times dh$$

Integrating both side

$$P = \int dF$$

$$= \int \rho g h \times b \times dh$$

$$= \rho g \int b \times h \times dh$$

we know that

$$\int b \times h \times dh = \int h \times dA$$

= moment of the strip from the free surface

Total moment of the body from free surface

$$= \int h \times dA$$

$$= A\bar{h}$$

$$\boxed{F = \rho g A \bar{h}}$$

Dt 7.02.2020

Center of Pressure (h^*)

It is calculated by using the principle of moments. which state that the moment of the resultant force about an axis is equal to the sum of moments of the components about the same axis

moment of force dF acting on a strip about free surface of liquid

$= dF \times h$

$= \rho g h \times b \times dh \times h$

sum of moments of ~~all~~ on such forces about free surface of liquid

$$= \int \rho g h \times b \times d \times h$$

$$= \rho g \int b.h.dh.h$$

$$= \rho g \int b h^2 dh$$

$$= \rho g \int h^2 dA$$

$\Rightarrow I_0$ = moment of inertia of the surface about free surface of liquid

Sum of moment about free surface

$= \rho g I_0$

The moment of force F about free surface

$= F \times h^*$

According to prinsiple of moments:

$\rho g I_0 = F \times h^*$

$\Rightarrow \rho g A\bar{h} \times h^* = \rho g I_0$

$\Rightarrow h^* = \dfrac{\rho g I_0}{\rho g A\bar{h}}$

$$\boxed{h^* = \frac{I_0}{A\bar{h}}}$$

According to parallel axis therom :

$I_0 = I_G + A\bar{h}^2$

$h^* = \dfrac{I_0}{A\bar{h}}$

$= \dfrac{I_G + A\bar{h}^2}{A\bar{h}}$

$$\boxed{h^* = \frac{I_G}{A\bar{h}} + \bar{h}}$$

Rectangle

moment of inertia about base $\dfrac{bd^3}{3}$

moment of inertia about an axis passing through C_G and parallel to base $\dfrac{bd^3}{12}$

Triangle

About base $= \frac{bh^3}{12}$

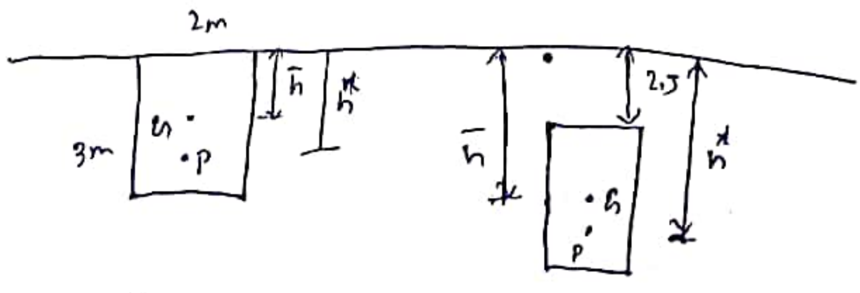
About C_g $= \frac{bh^3}{36}$

Circle $= \frac{\pi d^4}{64}$

Q A rectangular plain surface is 2m wide and 3 m depth it lies in vertical plan on water determine the total pressure and position of center of pressure on the plain surface when it upper edge is horizontal and ① coinsides with water surface

② 2.5 m below the free water surface.

$\bar{h}$ = Distance of C_G from free surface



Case I Case II

Case I coinsides with water surface

$F = \rho g A \bar{h}$

$= 1000 \times 9.81 \times 2 \times 3 \times 1.5$

$= 88290$ N

Case II

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

$$F = \rho g h A \bar{h}$$

$$= 1000 \times 9.81 \times 6 \times 4$$

$$= 235440 \text{ N}$$

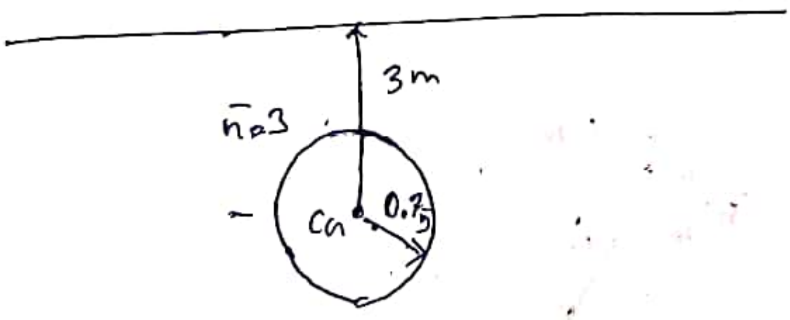
$$I_G = \frac{bd^3}{12}$$

$$= \frac{2 \times (3)^3}{12} = 4.5 \ m^4$$

$$h^* = \frac{4.5}{6 \times 4} + 4$$

$$h^* = 4.1875 \ m$$

Q Determine the total pressure on a circular plate of diameter 1.5 m which is placed vertically in water in such a way that the center of the plate is 3 m below the free surface of water find the position of center of pressure also



$$F = \rho g A \bar{h}$$

$$= 1000 \times 9.81 \times \frac{\pi}{4} d^2 \times 3$$

$$= 52007.102 \text{ N}$$

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

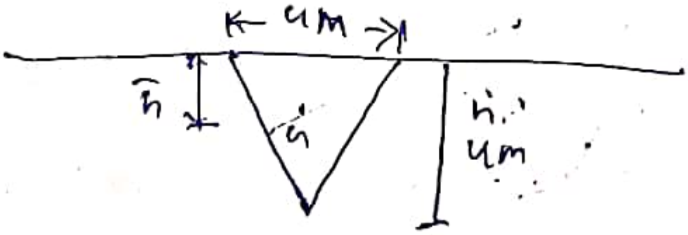
$$I_G = \frac{\pi d^4}{64}$$

$$= 0.2485 \ m^4$$

$$h^* = \frac{0.2485}{1.7671 \times 3} + 3$$

$$= 3.0468 \ m$$

Q Determine the total pressure and center of pressure on an isoscale triangle plate of base 4m and altitude 4m when it is immersed vertically in an oil of specific gravity 0.9 the base of the plate coinsides with the free surface of the oil



$$\bar{h} = \frac{h}{3} = \frac{4}{3} = 1.33 \ m$$

$F = \rho g A \bar{h}$

$= 0.9 \times 10^3 \times 9.81 \times \frac{1}{2} b \times h \times 1.33$

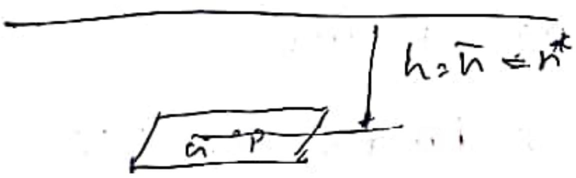
$= 93940.56$ N

$h^* = \frac{I_G}{A \bar{h}} + \bar{h}$

$I_G = \frac{bh^3}{36} = 7.11 \ m^4$

$h^* = \frac{7.11}{8 \times 1.33} + 1.33$

$= 1.998$ m



$F = \rho g A h$

Dt 14.02.2020

Orifices and mouth piece

vena - contracta :

Orifices :

It is a small opening of any cross section (circular, triangular, rectangular, etc.) on the side or on the bottom of a tank through which a fluid is flowing.

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mouth piece

It is a short length of a pipe which is two or three times of diameter in length fitted in a tank or vessel containg the fluid.

Classification of orifice

① Small orifice and large orifice

② circular, triangular or rectangular, etc

③ sharp edged orifice

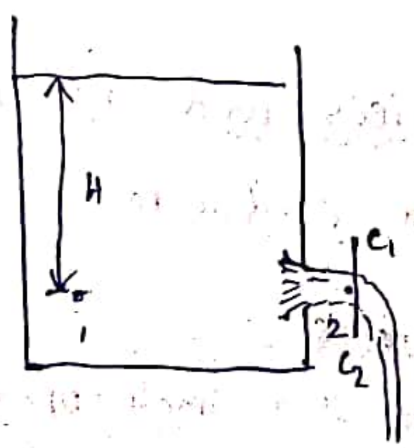
bell mouth orifice

④ Free discharging orifice

Drowned or submerged orifice

Flow through an orifice

vena contracta



It a distance of $\frac{1}{2}$ of diameter of the orifice at this section the streamline are straight and parallel to each other and perpendicular to the plane of the orifice ~~beon~~ beyound this section the jet diverges and it attracted in the downward direction dues to gravity

consider two section 1 and 2 inside the tank and at the vena contracta respectively

Appling Bernoulli's eq^n at ① and ②

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\Rightarrow \frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} \quad (\text{as } Z_1 = Z_2)$$

$$\Rightarrow H + 0 = 0 + \frac{V_2^2}{2g}$$

$$\Rightarrow \boxed{V_2 = \sqrt{2gH}}$$

Imp **Hydrolic coefficent**

Coefficent of velocity

It is defined as the actual velocity of a jet of liquid at vena contracta and the theoretical velocity of the jet

$$C_V = \frac{\text{Actual velocity}}{\text{Theorital velocity}}$$

$$= \frac{V}{\sqrt{2gh}}$$

It is denoted by CV and it's value varies from

0.95 to 0.99

Coefficent of contraction

It is defind as the ratio of the area of jets at vena contracta to the area of orifice

$$C_c = \frac{\text{Area of vena contracta}}{\text{Area of orifice}}$$

$$= \frac{a_c}{a}$$

The value of C_c varies from 0.61 - 0.69

Coefficent of discharge

It is defind as the ratio of actual discharge form an orifice to the therotical discharge from the orifice

$$C_d = \frac{\text{Actual discharge}}{\text{Therotical discharge}}$$

$$= \frac{\text{Actual area} \times \text{Actual velocity}}{\text{Therotical area} \times \text{therotical velocity}}$$

$$= C_c \times C_v$$

The value of C_d varies from 0.61 - 0.65

Q The head of water over an orifice of diameter 40mm is 10 m find the actual discharge and actual velocity of the jet at vena contracta

$C_d = 0.6 \quad C_v = 0.98$

Given

$D = 0.04 \, m$

$H = 10 \, m$

$a = \frac{\pi}{4} (D)^2$

$= 0.00125 \, m^2$

Actual discharge

= Actual area × actual velocity

$V_2 = \sqrt{2gH}$

$= \sqrt{2 \times 9.81 \times 10}$

$= 14.007 \, m/s$

$C_v = \frac{V}{\sqrt{2gH}}$

$V = C_v \times \sqrt{2gH}$

$= 0.98 \times 14.007$

$= 13.726 \, m/s$

$$C_d = \frac{a_c \times V_1}{a \times V_2}$$

$$a_c = \frac{Cd \times a \times V_2}{V_1}$$

$$= \frac{0.6 \times 0.00125 \times 14.007}{13.726}$$

$$= 7.65 \times 10^{-4}$$

$$= 0.000765$$

$$Cd = \frac{\text{Actual discharge}}{0.0175}$$

Actual discharge $= 0.6 \times 0.0175$

$= 0.0105\ m^3/s$

Q The head of water over the center of the orifice of diameter 20 mm is 1 m the actual discharge through the orifice is 0.85 lit/s find the coeffient of discharge

$D = 20\ mm = 0.02$

$H = 1\ m$

Actual discharge $= 0.85 \times 10^{-3}$

$$a = \frac{\pi}{4}(D)^2$$

$= 3.141 \times 10^{-4}\ m^2$

$v_j = \sqrt{2gH}$

$= \sqrt{2 \times 9.81 \times 1}$

$= 4.429 \ m/s$

Th. discharge $= 3.141 \times 10^{-4} \times 4.429$

$= 1.391 \times 10^{-3} \ m^3/s$

$$cd = \frac{0.85 \times 10^{-3}}{1.391 \times 10^{-3}}$$

$= 0.611$

Dt 18.2.2020

Surface tension

It is defined as the tensile force acting on the surface of a liquid in contact with gas or on the surface between two immisible liquid such that the contant surface behave like a membrane

Surface tension on liquid droplets

Consider a small spherical droplet of a liquid of radius 'r'

Let σ = surface tension of the liquid

p = pressure intensity in side the droplet

d = diameter of droplet.

Tensile force due to surface tension is equal to

$\sigma \times \pi d$

Pressure force on the area = $p \times \frac{\pi}{4} d^2$

For equlibrium

$$\sigma \times \pi d = p \times \frac{\pi}{4} d^2$$

$$p = \frac{4\sigma}{d}$$

Surface tension on a hollow bubble

$$2(\sigma \times \pi d) = p \times \frac{\pi}{4} d^2$$

$$\sigma \times \pi d = p \times \frac{\pi d^2}{8}$$

$$p = \frac{8\sigma}{d}$$

Q The surface tension of water in contact in air at 20°C is 0.0725 N/m the pressure inside a droplet of water is 0.02 N/cm² greater then the outside pressure calculate to diameter of droplet of water

Give

$\sigma = 0.0725$

$P = 0.02 \times 10^4$

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$$d = \frac{4\sigma}{p}$$

$$= 1.45 \times 10^{-3}$$

$$= 0.00145 \ m$$

Q Find the surface tension is a soap bubble of 40mm diameter when the inside pressure is 2.5 N/m² above atmospheric pressure

Give

$p = 2.5 N/m^2$

$d = 40 \ mm = 0.04 \ m$

$$\sigma = \frac{pd}{8}$$

$$= 0.0125 \ N/m$$

Q The pressure outside the droplet of water of diameter 0.04mm is 10.32 N/cm². Calculate the pressure within the droplet if surface tension is 0.0725 N/m

Give

$d = 0.04 \ mm = 4 \times 10^{-6} \ m$

$p = 10.32 \ N/cm^2 = 10.32 \times 10^4 \ N/m^2$

$\sigma = 0.0725 \ N/m$

$$P_s = \frac{4\sigma}{d} = 72500$$

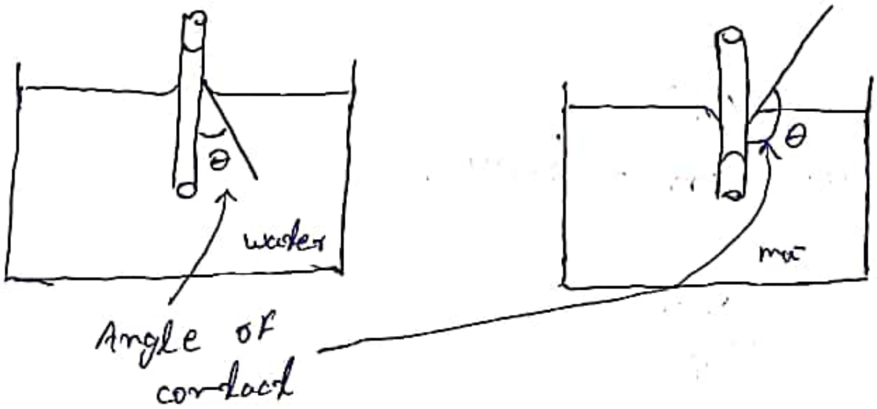
$$P_T = P_s + P$$

$$= 175700 \ \text{(wrong)}$$

Dt 28.02.2020

Capillarity

It is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjusent generally level of liquid when the tube is held vertically in the liquid



Expraction for capillarity rise

$$h = \frac{4\sigma}{\rho g d}$$

Expraction for capillary fail

$$h = \frac{4\sigma\cos\theta}{\rho g d}$$

wher

h = hight of the liquid in the tube

σ = surface tension of liquid

θ = angle of contact between liquid of glass tube

Q Calculate the capillary rise in a glass tube of 2.5 mm diameter when immersed vertically in

(1) water

(2) mercury

Take surface tension 0.0725 N/m for water and 0.052 N/m for mercury. Angle of contact is 130°

Ans

Given

$d = 2.5\ mm = 0.0025$ $\theta = 130^\circ$

$\sigma = 0.0725$ water

$\sigma = 0.052$ mercury

water

$$h = \frac{4\sigma}{\rho g d}$$

$= 0.011\ m$

mercury

$$h = \frac{4\sigma\cos\theta}{\rho g d}$$

$= -0.0004\ m$

Q Calculate the capillary effect in a glass tube of 4 mm diameter when immersed in water and mercury the values of water and mercury with air are 0.073575 N/m and 0.51 N/m respectively the angle of contact for mercury is 130° take density of water 998 kg/m³

Q The capillary rise in a glass tube is not to exceed 0.2 mm of water determine its minimum size given that surface tension of water in contact with air is 0.0725 N/m

$$d = \frac{4\sigma}{\rho g h}$$

$= 1.47$

Newtons law of viscosity

It state that the shear stress in a ~~fluid~~ fluid element layer is directly proportional to shear strain

$$\tau \propto \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy}$$

μ = viscosity

τ = shear stress

ideal fluid:

A fluid which is incomprasible and is having no viscosity is known as ideal fluid

Real fluid

A fluid which pos viscosity is known as real fluid

Newtonion fluid

obays Newtons law of fluid

Non Newtonion fluid

A plate 0.025 mm distance from a fixed plate move at 60 cm/s and required a force 6 P 2 N/area to maintain its speed. Determine the fluid viscocity between the plate

$dy = 0.025\ mm = 2.5 \times 10^{-5}$

$du = (60 - 0) = 60\ cm/s = 0.6 m$

$\tau = 2\ N/m2$

$$\mu = \frac{du}{dy} \times \tau$$

$= 12000 \quad N\ sec/m^2$

Dyen sec/cm^2

1 Dyen sec per cm^2 is equal to 1 poise

1 Poise $= \frac{1}{10}$ N sec/m^2

1 scend poise $= \frac{1}{100}$ poise

Kinematic viscocity

It is defined as the ratio between dynamic viscocity and density of fluid

$$\boxed{\nu = \frac{\mu}{\rho}}$$

unit SI unit m^2/sec

CGS unit cm^2/sec = 1 stoke

$\frac{du}{dy}$ is unown as velocity ~~d~~ gradient

Find the Kinematic viscocity of an oil having density 9.81 kg/m3 the shear ~~stres~~ stress at a point ~~o~~ in oil is 0.2452 N/m2 and velocity gradient at that point is 0.2/sec

$$\nu = \frac{\mu}{\rho}$$

$$\mu = \frac{dy}{du} =$$

$$= \frac{1}{0.2} \times 0.2452$$

$$= 1.226 \text{ N sec/m}^2$$

$$\nu = \frac{1.226}{9.81 \times 10^3}$$

$$= 0.1249 \times 10^{-3} \text{ m}^2/\text{sec}$$

$$= \frac{0.1249 \times 10^{-3} \times 10^4}{\cancel{1}} \text{ cm}^2/\text{sec}$$

$$= 1.249$$

Notches and weirs

Dt 3.03.2020

Notch

It is a device used for measuring the rate of flow or a liquid through a small channel or tank

* It may be defined as an opening in the side of a tank or a small channel in ~~so~~ such a way that the liquid surface in the tank or channel is below the top edge of the opening

weir

It is a concrid or masonary structure placed in an open channel over with the flow occures.

* It is generally in the form of vertical wall with a sharp edge at the top runing all the way across the open channel.

Classification of Notch and weire

-> According to shape of opening

1. Rectangluar notch
2. Triangular notch
3. Trapezoidal notch
4. stepped notch

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According to the effect of sides on the nappe

① notch with end contraction

② notch with out end contraction or ~~suppred~~ suppressed notch

According to the shape of opening

① Rectangular weir

② Triangular weir

③ Trapezoidal weir (Cipolletty weir)

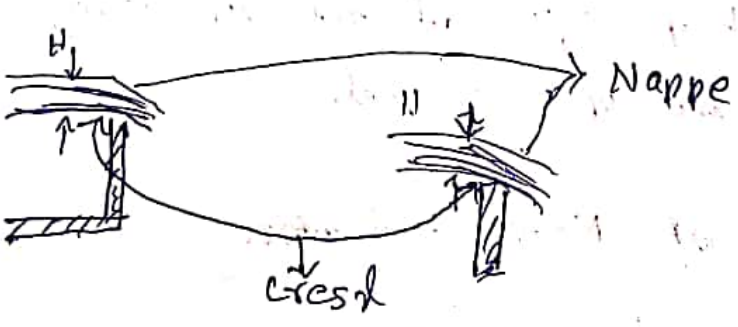
According to the shape of the crest

① Sharp crested weir

② Broad crested weir

③ narrow crested weir

④ ogee shaped weir



Discharge over a rectangular notch or weir

$$\boxed{Q = \frac{2}{3} C_d L \sqrt{2g} (H)^{3/2}}$$

L = Length of the notch or weir

H = heigh or head of water over the crest.

Q) Find the discharge of water flowing over a rectangular notch of 2 m length when the constant head over the notch is 300 mm take cd = 0.6

$Q = \frac{2}{3}(0.6)\, 2 \times \sqrt{2 \times 9.81}\, (0.3)^{3/2}$

$= 0.5822\ m^3/s$

Q) The head of water over a rectangular notch is 900 mm the discharge is 300 lit/sec find the length of the notch cd = 0.62

$Q = 300\ m^3/sec$

$H = 0.9$

$Q = 5.042\ L$

$L = 0.191\ m$

Q) Determine the height of a rectangular weir of length 6 m to be build across a rectangular channel the maximum depth of water on the up strem side of the weir is 1.8 m and discharge is 2000 lit per sec take cd = 0.6

Given

$L = 6\ m$

$H + h = 1.8\ m \Rightarrow h = 1.8 + H$

$$2 = \frac{2}{3} \times 0.6 \times 6 \sqrt{2 \times 9.81} \, (H)^{\frac{3}{2}}$$

$$(H)^{\frac{3}{2}} = \frac{2}{10.630}$$

$$H = (0.1881)^{\frac{2}{3}}$$

$$= 0.328 \text{ m}$$

$$h = 1.8 - H$$

$$= 1.8 - 0.328$$

$$= 1.472 \text{ m}$$

Discharge over a triangular notch or weir

$$\boxed{Q = \frac{8}{15} C_d \tan\frac{\theta}{2} \sqrt{2g} \, (H)^{5/2}}$$

Q1) Find the discharge over a triangular notch of angle 60° when the head over V notch is 0.3 m cd = 0.6

$$Q = \frac{8}{15} \, 0.6 \times \tan\left(\frac{60}{2}\right) \sqrt{2 \times 9.81} \times (0.3)^{5/2}$$

$$= 0.0403 \text{ m}^3/\text{s}$$

Q water flowes over a rectangular wire 1m wide At a depth of 150mm and after work passes through a ~~tr~~ triangular righ angle weir taking cd for rectangular and triangular as 0.62 and 0.59 respectively value find the depth of ~~o~~ over the triangluar weir

Given

$L = 1m$

$H = 0.15$

$cd = 0.62$

$cd = 0.59$ Δ

$$Q = \frac{2}{3} cd \times L \sqrt{2g} (H)^{\frac{3}{2}}$$

$$= 0.1063 \ m^3/s$$

$$Q = \frac{8}{15} cd \sqrt{2g} (H)^{\frac{5}{2}}$$

$$0.1063 = 1.3937 (H)^{\frac{5}{2}}$$

$$H = (0.0762)^{\frac{2}{5}}$$

$$= 0.357 \ m$$

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Dt 6.03.2020

Flow through pipes

Loss of energy in pipes

when a fluid is flowing through a pipe the fluid expances some resistance due to which some of the energy of fluid is lost

Loss of energy or head due to friction

Darcy - weisbach formula :

$$\boxed{h_f = \frac{4fLV^2}{d \cdot 2g}}$$

where

h_f = Loss of head due to friction

L = Length of pipe

V = mean velocity of flow

d = diameter of pipe

f = coefficent of friction which is a function of reynold number

$$f = \frac{16}{R_e} \quad (R_e < 2000)$$

$$f = \frac{0.079}{R_e^{1/4}} \quad (4000 < R_e < 10^6)$$

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Chezy's formula

$$\boxed{V = C\sqrt{mi}}$$

where

c = chezy's constant

m = hydrolic mean depth

$$m = \frac{A}{P} = \frac{\frac{\pi}{4}d^2}{\pi d} = \frac{d}{4}$$

It is defined as the ratio of area of flow to the wetted perimeter.

i = loss of head per unit length of pipe

$$= \frac{h_f}{L}$$

V = mean velocity

Q Find the head lost due to friction in a pipe of diameter 300 mm and length 50 meter through which water is flowing at a velocity of 3m/s using ① Darcy formula

② chezy's formula for which

C = 60

Take viscocity of water = 0.01 stock

$$\boxed{Re = \frac{V \times d}{\nu}}$$

where V = velocity

ν = viscocity

d = diameter

Given

$d = 0.3$ m

$L = 50m$

$v = 3m/s$

$\nu = 0.01$ stock $= 0.01\ cm^2/s = 0.01 \times 10^{-4}\ m^2/s$

(1) $C = 60$

$$Re = \frac{V \times d}{\nu}$$

$$= 900000$$

$$f = \frac{0.079}{Re^{1/4}} = 2.56 \times 10^{-3}$$

$$h_f = \frac{4 f L v^2}{d \cdot 2g}$$

$$= 0.764\ m$$

(2) $$m = \frac{d}{4}$$

$$= \frac{0.3}{4}$$

$$= 0.075\ m$$

$$V = C\sqrt{mi}$$

$$V = C\sqrt{m \times \frac{h_f}{L}}$$

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$$v = 16.431 \sqrt{\frac{hf}{L}}$$

$$3 = 16.431 \frac{\sqrt{hf}}{\sqrt{50}}$$

$$3 = 16.431 \frac{\sqrt{hf}}{7.071}$$

$$\sqrt{hf} = \frac{3}{16.431} \times 7.071$$

$$hf = (1.291)^2$$

$$= 1.66 \text{ m}$$

Q find the diameter of pipe of length 2000 m when the rate of flow of water through the pipe is 200 lit/sec and the head lost due to friction is 4 m take the value of $C = 50$

Given

$L = 2000 m$

$Q = 200 \text{ lit/sec} \quad \frac{200}{1000} = 0.2 \text{ m/sec}$

$= A.V = \frac{\pi}{4} d^2 \times V$

$= V = \frac{0.2}{\frac{\pi}{4} d^2} =$

$$\frac{0.256}{d^2} = 50 \sqrt{0.25d \times 2 \times 10^{-3}}$$

$$= 1.118 \sqrt{d}$$

$$\frac{1}{d^2} = \frac{1.118\sqrt{d}}{0.256}$$

$$= 4.367\sqrt{d}$$

$$\frac{1}{d^2}$$

$$\frac{1}{d^4} = 19.070689\, d$$

$$d^5 = \frac{1}{19.070689}$$

$$d = \sqrt[5]{0.0524}^{1/5}$$

$$= 0.554 \text{ m}$$

A crude oil of kinematic viscosity 0.4 stoke is flowing through a pipe of diameter 300 mm at the rate of 300 lit/sec. find the head lost due to friction for a length of 50m of the pipe

$\nu = 0.4$ stoke $= 0.4 \times 10^{-4}\ m^2/s$

$d = 300$ mm $= 0.3$ m

$Q = 300$ lit/sec $= 0.3\ m/sec$

$L = 50$ m

solution

$Q = A \times V$

$= \frac{\pi}{4} d^2 \times V$

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$$0.3 = \frac{\pi}{4} \times (0.3)^2 \times V$$

$$V = 4.24 \text{ m/s}$$

$$Re = \frac{V \times d}{\nu}$$

$$= 31800$$

$$f = \frac{0.079}{Re^{1/4}}$$

$$= 5.91 \times 10^{-3}$$

$$h_f = \frac{4 f L V^2}{d \cdot 2g}$$

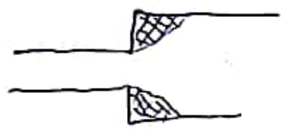
$$= 3.857 \text{ m}$$

Dt 13.03.20

minor energy lose

① Loss of head due to sudden enlargement

$$\boxed{h_e = \frac{(V_1 - V_2)^2}{2g}}$$



② Loss of head due to sudden contraction.

$$\boxed{h_c = 0.5 \frac{V_2^2}{2g}}$$

③ Loss of head at the entrance of a pipe

$$\boxed{h_i = 0.5 \frac{V^2}{2g}}$$

④ Loss of head at the exit of a pipe

$$\boxed{h_o = \frac{V^2}{2g}}$$

Q1 Determine the rate of flow of water through a pipe of diameter 20 cm and length 50 m when 1 end of the pipe is connected to a tank and other end of the pipe is open to the atmospher the pipe is horizontal and hight of water in the tank is 4 m above the centero of the pipe. concider all minor losse and take $f = 0.009$ in the darcy formula

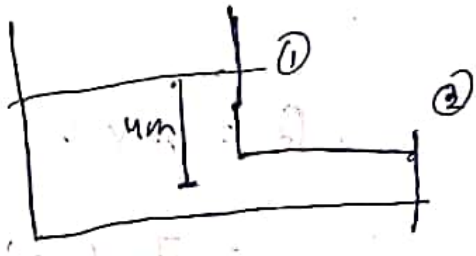
Given

$d = 0.2\,m$

$L = 50\,m$

$H = 4\,m$

$f = 0.009$



Taking two section at first in the starting of water in the tanke and other is at the pipe end

Applying Bernoulli's eqn at section 1 and 2

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + \text{All losses.}$$

$$\Rightarrow \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_i + h_f$$

$$\Rightarrow 0 + 0 + 4 = 0 + \frac{V_2^2}{2g} + 0 + 0.5\frac{V_2^2}{2g} + \frac{4fLV^2}{2g}$$

$$4 = \frac{V_2^2}{2g} + \frac{0.5V_2^2}{2g} + \frac{4fLV^2}{2g}$$

$$V_2^2 + 0.5V_2^2 + 4fLV_2^2 = 78.48$$

$$V_2^2 + 0.5V_2^2 + 1.8V_2^2 = 78.48$$

$$3.3V_2^2 = 78.48$$

$$V_2^2 = \frac{78.48}{3.3} = 23.7818$$

$V_2 = \sqrt{283.7818}$

$= 4.8766 \ m/s$ (2.4)

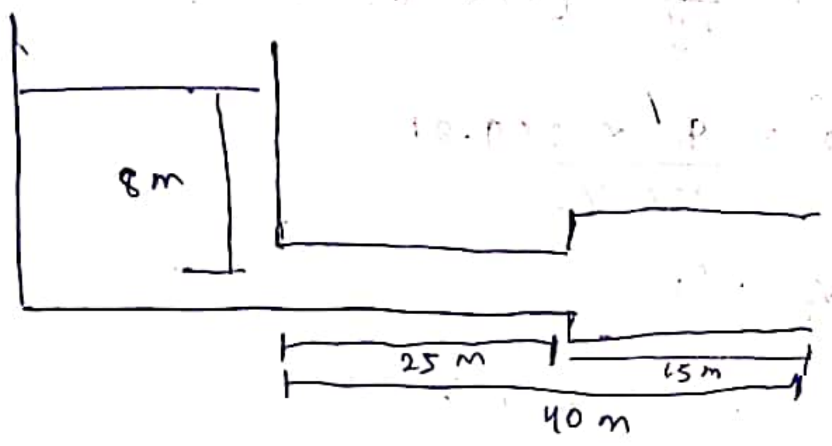
$Q = A \times V$

$= \frac{\pi}{4} d^2 \times V$

$= \frac{\pi}{4} \times (0.2)^2 \times 4.8766$

$= 0.15$ (0.075)

Q A horizontal pipe line 40 m long is connected to a water tank at one end and discharges freely into the atmosphere at the other end for the first 25 m of it's length from the tank the pipe is 150 mm diameter and it's diameter is suddly enlarge to 300 mm the hight of water in the tank is 8 m above the center of the pipe. considering all losses of head which occure determine the rate of flow take $f = 0.01$ for both section of pipe.



Appling bernoulli's eq^n at section ① and ②

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_i + h_{f_1} + h_e + h_{f_2}$$

$$z_1 = \frac{V_2^2}{2g} + \frac{0.5 V_1^2}{2g} + \frac{(V_1 - V_2)^2}{2g} + \frac{4fLV_1^2}{d_1 \cdot 2g} + \frac{4fLV_2^2}{d_2 \cdot 2g}$$

$$8 = \frac{V_2^2}{2g} + \frac{0.5 (4V_2)^2}{2g} + \frac{4fL(4V_2)^2}{d_1\, 2g} + \frac{(3V_2)^2}{2g} + \frac{4fLV_2^2}{d_2 gh}$$

$A_1 V_1 = A_2 V_2$

$0.017 V_1 = 0.070 V_2$

$$V_1 = \frac{0.070}{0.017} V_2$$

$V_1 = 4.011\ V_2$

$$8 = \frac{V_2^2}{2g}\left(1 + 0.5(4)^2 + \frac{4fL\,16}{d_1} + 9 + \frac{4fL}{d_2}\right.$$

$$8 = \frac{V_2^2}{2g}(126.667)$$

$$V_2^2 = \frac{8 \times 2 \times 9.81}{126.667}$$

$$V_2 = \sqrt{1.239}$$

$$= 1.113 \ m^2/s$$

$$Q = \underline{A_2 V_2}$$

$$= 0.0786 \ m^3/s$$

▶ 11.5 HYDRAULIC GRADIENT AND TOTAL ENERGY LINE

The concept of hydraulic gradient line and total energy line is very useful in the study of flow of fluids through pipes. They are defined as :

11.5.1 Hydraulic Gradient Line. It is defined as the line which gives the sum of pressure head $\left(\frac{p}{w}\right)$ and datum head (z) of a flowing fluid in a pipe with respect to some reference line or it is the line which is obtained by joining the top of all vertical ordinates, showing the pressure head (p/w) of a flowing fluid in a pipe from the centre of the pipe. It is briefly written as H.G.L. (Hydraulic Gradient Line).

11.5.2 Total Energy Line. It is defined as the line which gives the sum of pressure head, datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line. It is also defined as the line which is obtained by joining the tops of all vertical ordinates showing the sum of pressure head and kinetic head from the centre of the pipe. It is briefly written as T.E.L. (Total Energy Line).

Problem 11.22 *For the problem 11.16, draw the Hydraulic Gradient Line (H.G.L.) and Total Energy Line (T.E.L.).*

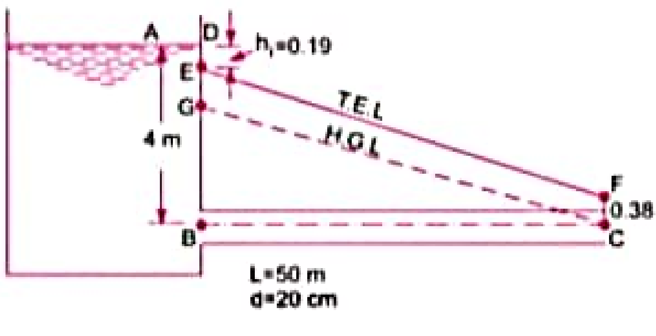
Solution. Given :

$$L = 50 \text{ m}, d = 200 \text{ mm} = 0.2 \text{ m}$$
$$H = 4 \text{ m}, f = .009$$

Velocity, V through pipe is calculated in problem 11.16 and its value is $V = 2.734$ m/s

Now, h_i = Head lost at entrance of pipe

$$= 0.5 \frac{V^2}{2g} + \frac{0.5 \times 2.734^2}{2 \times 9.81} = 0.19 \text{ m}$$



$$= \frac{4 \times f \times L \times V^2}{d \times 2g} = \frac{4 \times 0.009 \times 50 \times (2.734)^2}{0.2 \times 2 \times 9.81} = 3.428 \text{ m.}$$

(*a*) **Total Energy Line (T.E.L.).** Consider three points, *A*, *B* and *C* on the free surface of water in the tank, at the inlet of the pipe and at the outlet of the pipe respectively as shown in Fig. 11.8. Let us find total energy at these points, taking the centre of pipe as reference line.

1. Total energy at $A = \frac{p}{\rho g} + \frac{V^2}{2g} + z = 0 + 0 + 4.0 = 4$ m
2. Total energy at B = Total energy at $A - h_i = 4.0 - 0.19 = 3.81$ m
3. Total energy at $C = \frac{p_c}{\rho g} + \frac{V_c^2}{2g} + z_c = 0 + \frac{V^2}{2g} + 0 = \frac{2.734^2}{2 \times 9.81} = 0.38$ m.

Hence total energy line will coincide with free surface of water in the tank. At the inlet of the pipe, it will decrease by h_i (= 0.19 m) from free surface and at outlet of pipe total energy is 0.38 m. Hence in Fig. 11.8,

(*i*) Point *D* represents total energy at *A*

(*ii*) Point *E*, where $DE = h_i$, represents total energy at inlet of the pipe

(*iii*) Point *F*, where $CF = 0.38$ represents total energy at outlet of pipe.

Join *D* to *E* and *E* to *F*. Then *DEF* represents the total energy line.

(*b*) **Hydraulic Gradient Line (H.G.L.).** H.G.L. gives the sum of $(p/w + z)$ with reference to the datum-line. Hence hydraulic gradient line is obtained by subtracting $\frac{V^2}{2g}$ from total energy line. At outlet of the pipe, total energy $= \frac{V^2}{2g}$. By subtracting $\frac{V^2}{2g}$ from total energy at this point, we shall get point *C*, which lies on the centre line of pipe. From *C*, draw a line *CG* parallel to *EF*. Then *CG* represents the hydraulic gradient line.

Problem 11.23 *For the problem 11.17, draw the hydraulic gradient and total energy line.*

Solution. Refer to problem 11.17.

Given: $L_1 = 25$ m, $d_1 = 0.15$ m

$L_2 = 15$ m, $d_2 = 0.3$ m, $f = .01$, $H = 8$ m

The velocity V_2 as calculated in problem 11.17 is

$$V_2 = 1.113 \text{ m/s}$$

$$V_1 = 4V_2 = 4 \times 1.113 = 4.452 \text{ m/s}$$

The various head losses are $h_i = 0.5 \times \frac{V_1^2}{2g} = \frac{0.5 \times 4.452^2}{2 \times 9.81} = 0.50$ m

$$h_{f_1} = \frac{4f \times L_1 \times V_1^2}{d_1 \times 2g} = \frac{4 \times .01 \times 25 \times (4.452)^2}{0.15 \times 2 \times 9.81} = 6.73 \text{ m}$$

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(4.452 - 1.11)^2}{2 \times 9.81} = 0.568 \text{ m}$$

$$h_{f_2} = \frac{4 \times f \times L_2 \times V_2^2}{d_2 \times 2g} = \frac{4 \times .01 \times 15 \times (1.113)^2}{0.3 \times 2 \times 9.81} = 0.126 \text{ m}$$

$$h_o = \frac{V_2^2}{2g} = \frac{1.113^2}{2 \times 9.81} = 0.063 \text{ m}$$

Also $$V_1^2/2g = \frac{4.452^2}{2 \times 9.81} = 1.0 \text{ m.}$$

Total Energy Line

(*i*) Point A lies on free surface of water.

(*ii*) Take $AB = h_i = 0.5$ m.

(*iii*) From B, draw a horizontal line. Take BL equal to the length of pipe, *i.e.*, L_1. From L draw a vertical line downward.

(*iv*) Cut the line $LC = h_{f_1} = 6.73$ m.

(*v*) Join the point B to C. From C, take a line CD vertically downward equal to $h_e = 0.568$ m.

(*vi*) From D, draw DM horizontal and from point F which is lying on the centre of the pipe, draw a vertical line in the upward direction, meeting at M. From M, take a distance $ME = h_{f_2} = 0.126$. Join DE.

Then line $ABCDE$ represents the total energy line.

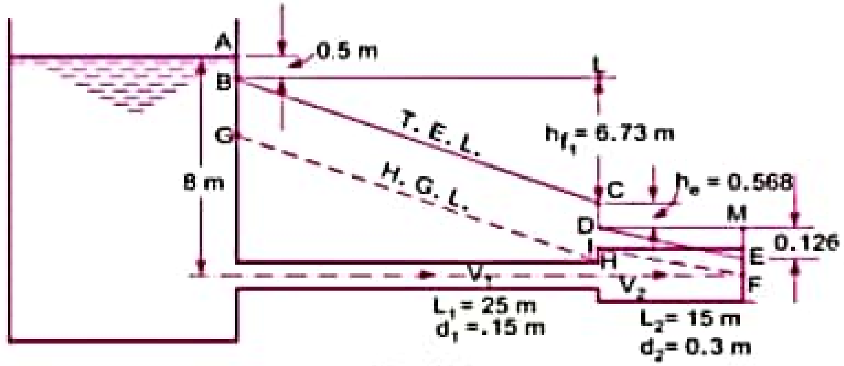


Fig. 11.9

Hydraulic Gradient Line (H.G.L.)

(*i*) From B, take $BG = \frac{V_1^2}{2g} = 1.0$ m.

(*ii*) Draw the line GH parallel to the line BC.

(*iii*) From F, draw a line FI parallel to the line ED.

(*iv*) Join the point H and I.

Then the line $GHIF$ represents the hydraulic gradient line (H.G.L.).

Problem 11.24 *For Problem 11.18, draw the hydraulic gradient and total energy line.*

Solution. Refer to Problem 11.18.

Given : $d = 300$ mm $= 0.3$ m

$L = 400$ m, $Q = 300$ litres/s $= 0.3$ m^3/s

$f = .008$

Let $\quad H_1 = 50$ m. But $H_1 - H_2 = 40.537$ m (Calculated in Problem 11.18)

$\therefore \quad H_2 = 50 - 40.537 = 9.463$ m.

The calculated losses are :

(*i*) $h_i = 0.459$ m $\quad$ (*ii*) $h_{f_1} = 39.16$ m

(*iii*) $h_o = 0.918$ m

(*a*) T.E.L.

(*i*) Point *A* is on the free surface of water in 1st tank. From *A*, take $AB = h_i = 0.459$ m.

(*ii*) Draw a horizontal line *BF*. Take *BF* equal to the length of pipe. From *F*, draw a vertical line in the downward direction. Cut $FC = h_{f_1} = 39.16$ m.

(*iii*) Join *BC*. From *C* take $CD = h_0 = 0.918$ m. The point *D* should coincide with free surface of water in 2nd tank. Then line *ABCD* is the total energy line.

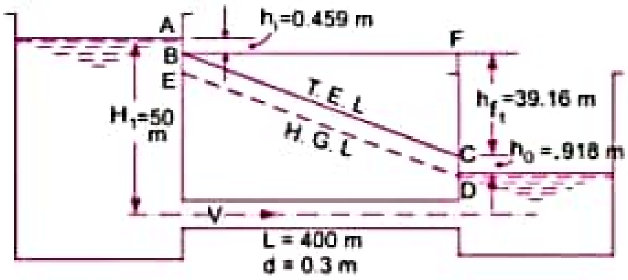


Fig. 11.10

(*b*) H.G.L. From *D*, draw a line *DE* parallel to line *BC*. Then *DE* is the H.G.L.

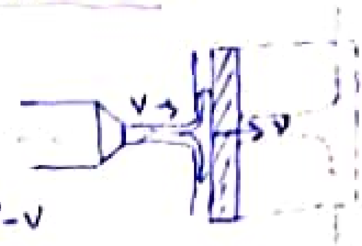
Or

From *B*, take $BE = \dfrac{V^2}{2g} = 0.918$ m and from *E* draw a line *ED* parallel to *BC*. The point *D* should coincide with free surface of water in the 2nd tank. Then line *ED* represents the H.G.L.

(c) IMPACT OF JET ON A MOVING FLAT PLATE (VERTICAL)

Let, V = velocity of the jet

v = velocity of the plate



Velocity of the jet relative to the plate = $V-v$

We may consider as though the plate is at rest & that the jet is moving with a velocity $(V-v)$ relative to the plate.

∴ Force exerted by the jet on the plate

$$F = \rho a (V-v)^2 \text{ Newton}$$

In this case, since the point of application of the force moves, work is done by the jet.

Workdone by the jet on the plate per second

$$= Fv = \rho a (V-v)^2 v \text{ Nm/s or, Joule/sec.}$$

(d) IMPACT OF JET ON A MOVING FLAT PLATE (INCLINED)

Let the velocity of the jet & the vane be V & v in the same direction. Let the angle between the jet & the plate be θ. In this case mass of liquid striking the plate per second = $\rho a (V-v)$

Relative velocity normal to the plate before impact

$$= (V-v)\sin\theta$$

Relative velocity normal to the plate after impact

$$= 0$$

∴ Force exerted by the jet normal to the plate

$$F = \rho a (V-v)[(V-v)\sin\theta - 0]$$

$$\Rightarrow F = \rho a (V-v)^2 \sin\theta \text{ Newton}$$

This force F acting normal to the plate can be resolved into components F_x & F_y in the direction of motion of plate & perpendicular to the direction of motion of the plate.

$F_x = \rho a (V-v)^2 \sin^2\theta$ & $F_y = \rho a (V-v)^2 \sin\theta\cos\theta$

∴ Workdone by jet per second = $F_x v = \rho a (V-v)^2 v \sin^2\theta$.

∴ Force exerted by the jet normal to the plate

F = mass striking the plate /sec × Change in velocity normal to the plate

$= M(V\sin\theta - 0) = \rho a V \cdot V\sin\theta$

$\Rightarrow \boxed{F = \rho a V^2 \sin\theta}$ Newton.

* In the above two cases the workdone by the jet on the plate is zero since the point of application of the force doesn't move.

4th sem
Mech. Engg.
FM & HM

Impact of Jet

Notes by
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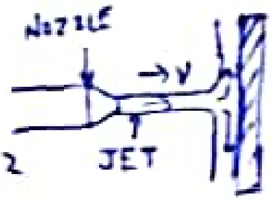
Introduction to Jet → A jet of water issuing from a nozzle has a velocity & hence it possesses a K.E.. If this jet strikes a plate then it is said to have an impact on the plate. The jet will exert a force on the plate which it strikes. This force is called dynamic force exerted by the jet. This force is due to the change in momentum of the jet as a consequence of the impact. This force is equal to the rate of change of momentum. i.e. mass striking/sec × change in velocity.

Impact of jet on a Stationary Flat plate (vertical)

Consider a jet of water impinging normally on a flat plate at rest.



Let, a = Cross-sectional area of the jet in m^2

V = velocity of the jet in m/s

M = Mass of water striking the plate per second.

$\therefore M = \rho a V$ kg/sec.

Where, ρ = density of water in kg/m3

Force exerted by the jet on the plate

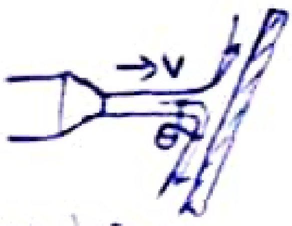
F = change in momentum per second

= mass striking the plate/sec × change in velocity

$= M(V-0) = MV = \rho aV.V$

$\Rightarrow \boxed{F = \rho a V^2}$ Newton

Impact of jet on a Stationary Flat Plate (inclined)



Let, θ = Angle between jet & plate

Velocity component normal to the plate before impact $= V\sin\theta$

Velocity component normal to the plate after impact $= 0$.

17.2.2 Force Exerted by a Jet on Stationary Curved Plate

(A) Jet strikes the curved plate at the centre. Let a jet of water strikes a fixed curved plate at the centre as shown in Fig. 17.3. The jet after striking the plate, comes out with the same velocity if the plate is smooth and there is no loss of energy due to impact of the jet, in the tangential direction of the curved plate. The velocity at outlet of the plate can be resolved into two components, one in the direction of jet and other perpendicular to the direction of the jet.

Component of velocity in the direction of jet = $-V \cos \theta$.

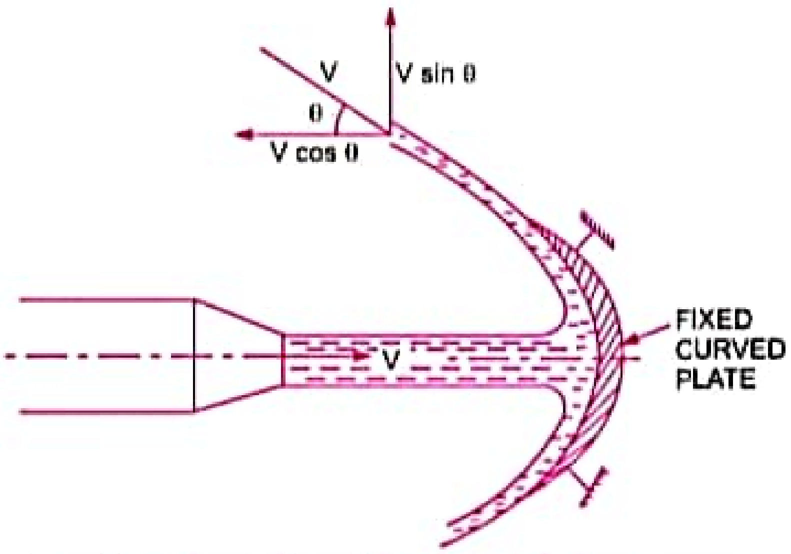


Fig. 17.3 *Jet striking a fixed curved plate at centre.*

(–ve sign is taken as the velocity at outlet is in the opposite direction of the jet of water coming out from nozzle).

Component of velocity perpendicular to the jet = $V \sin \theta$

Force exerted by the jet in the direction of jet,

$$F_x = \text{Mass per sec} \times [V_{1x} - V_{2x}]$$

where V_{1x} = Initial velocity in the direction of jet = V

V_{2x} = Final velocity in the direction of jet = $-V \cos \theta$

$\therefore$ $$F_x = \rho aV[V - (- V \cos\theta)] = \rho aV[V + V\cos\theta]$$
$$= \rho aV^2[1 + \cos\theta] \quad \text{...(17.5)}$$

Similarly, $$F_y = \text{Mass per sec} \times [V_{1y} - V_{2y}]$$

where V_{1y} = Initial velocity in the direction of $y = 0$

V_{2y} = Final velocity in the direction of $y = V \sin\theta$

$\therefore$ $$F_y = \rho aV[0 - V\sin\theta] = -\rho aV^2 \sin\theta \quad \text{...(17.6)}$$

–ve sign means that force is acting in the downward direction. In this case the angle of deflection of the jet $= (180° - \theta)$ ...[17.6 (*A*)]

(B) Jet strikes the curved plate at one end tangentially when the plate is symmetrical. Let the jet strikes the curved fixed plate at one end tangentially as shown in Fig. 17.4. Let the curved plate is symmetrical about x-axis. Then the angle made by the tangents at the two ends of the plate will be same.

Let V = Velocity of jet of water,

θ = Angle made by jet with x-axis at inlet tip of the curved plate.

If the plate is smooth and loss of energy due to impact is zero, then the velocity of water at the outlet tip of the curved plate will be equal to V. The forces exerted by the jet of water in the directions of x and y are

$$F_x = (\text{mass/sec}) \times [V_{1x} - V_{2x}]$$
$$= \rho aV[V\cos\theta - (-V\cos\theta)]$$
$$= \rho aV[V\cos\theta + V\cos\theta]$$
$$= 2\rho aV^2 \cos\theta \quad \text{...(17.7)}$$
$$F_y = \rho aV[V_{1y} - V_{2y}]$$
$$= \rho aV[V\sin\theta - V\sin\theta] = 0$$

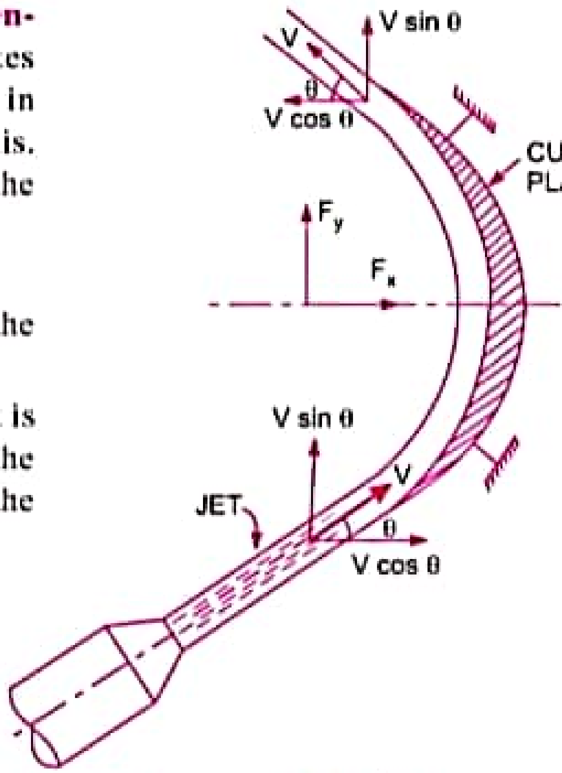


Fig. 17.4 *Jet striking curved fixed plate at one end.*

(C) Jet strikes the curved plate at one end tangentially when the plate is unsymmetrical. When the curved plate is unsymmetrical about x-axis, then angle made by the tangents drawn at the inlet and outlet tips of the plate with x-axis will be different.

Let θ = angle made by tangent at inlet tip with x-axis,

ϕ = angle made by tangent at outlet tip with x-axis.

The two components of the velocity at inlet are

$$V_{1x} = V\cos\theta \text{ and } V_{1y} = V\sin\theta$$

The two components of the velocity at outlet are

$$V_{2x} = -V\cos\phi \text{ and } V_{2y} = V\sin\phi$$

$\therefore$ The forces exerted by the jet of water in the directions of x and

$$F_x = \rho aV[V_{1x} - V_{2x}] = \rho aV[V\cos\theta - (-$$
$$= \rho aV[V\cos\theta + V\cos\phi] = \rho aV^2[\cos\theta + \cos\phi] \quad \text{...(17.8)}$$
$$F_y = \rho aV[V_{1y} - V_{2y}] = \rho aV[V\sin\theta - V\sin\phi]$$
$$= \rho aV^2[\sin\theta - \sin\phi]. \quad \text{...(17.9)}$$